

DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

M.A/M.Sc. 2nd Semester

Name of Programme	: M.A/M. Sc. Mathematics
Paper Type	: Theory
Paper Code	: OPE-006
Paper Title	: Fuzzy set theory and Mathematical Programming

Full Marks : 80

Pass Marks : 32

Duration: 3 Hours

The figures in the margin indicate full marks for the questions

Answer all the questions:

Answer any three of the following questions:

$10 \times 3 = 30$

1.
 - a) Prove that sum of two convex fuzzy sets are convex.
 - b) Let \tilde{P} and \tilde{Q} be two fuzzy sets defined on the universal set X ,
Prove that $|\tilde{P}| + |\tilde{Q}| = |\tilde{P} \cup \tilde{Q}| + |\tilde{P} \cap \tilde{Q}|$ 5+5=10
2. Consider the fuzzy set \tilde{A} , \tilde{B} and \tilde{C} defined on the interval $X = [0,10]$ of real number by the membership grade functions. $\tilde{A}(x) = \frac{1}{2x}$, $\tilde{B}(x) = \frac{x}{x+2}$, $\tilde{C}(x) = \frac{1}{1+10(x-2)}$

Determine the given mathematical formulas

i) $\tilde{A}^c, \tilde{B}^c, \tilde{C}^c$

ii) $\tilde{A} \cup \tilde{B}, \tilde{B} \cup \tilde{C}, \tilde{A} \cup \tilde{C}$

iii) $\tilde{A} \cup \tilde{B} \cup \tilde{C},$

iv) $\tilde{A} \cap \tilde{B} \cap \tilde{C}$

v) $\tilde{B} \cap \tilde{C}, \overline{\tilde{B} \cup \tilde{C}}$

OPE-006

3. Let \tilde{P} and \tilde{Q} be two fuzzy sets whose membership functions are defined as follows

$$\mu_{\tilde{P}}(x) = \begin{cases} 0, & \text{if } x \leq 2 \\ \frac{x-2}{2}, & \text{if } 2 < x \leq 4 \\ \frac{6-x}{2}, & \text{if } 4 < x \leq 6 \\ 0, & \text{if } x > 6 \end{cases}$$

$$\mu_{\tilde{Q}}(x) = \begin{cases} 0 & \text{if } x \leq -1 \\ \frac{x+1}{2}, & \text{if } -1 < x \leq 1 \\ \frac{3-x}{2}, & \text{if } 1 < x \leq 3 \\ 0 & \text{if } x > 3 \end{cases}$$

Find $\mu_{\tilde{P}+\tilde{Q}}$ and $\mu_{\tilde{P}-\tilde{Q}}$

4. What is defuzzification of a fuzzy set. If

$$\tilde{P} = \{(1,0), (2,0.5), (3,1), (4,1), (5,1), (6,0.75), (7,0.5), (8,0.2), (9,0), (10,0)\}$$

$$\tilde{Q} = \{(1,0), (2,0), (3,0), (4,0.33), (5,0.67), (6,1), (7,1), (8,1), (9,0.5), (10,0)\}$$

Find the defuzzified value of combination of \tilde{P} and \tilde{Q} using

- a) Centre of sums and
 - b) Centre of area
5. Explain the following with an example
- a) Product of two fuzzy sets,
 - b) Product of a fuzzy set with a crisp,
 - c) Difference of two fuzzy set,
 - d) Disjunctive sum of two fuzzy set,
 - e) Power of a fuzzy set.

Answer any three of the following questions:

6. Answer the following questions:

a) Find all the basic feasible solutions for the system

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

Also, name the basic and non-basic variables in the solutions obtained by you. 5

b) Solve graphically the following LPP

$$\text{Maximize } Z = 8x_1 - 2x_2$$

Subject to the constraints

$$x_1 - 3x_2 \geq -3$$

$$2x_1 + 3x_2 \geq -6$$

$$2x_1 + x_2 \leq 8$$

$$4x_1 - x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

5

7. Use Simplex method to solve the following LPP. 10

$$\text{Maximize } Z = 2x_1 + x_2$$

Subject to the constraints

$$3x_1 + 5x_2 \leq 15$$

$$6x_1 + 2x_2 \leq 24$$

$$x_1, x_2 \geq 0$$

8. Use big M method or two phase method to solve the following LPP. 10

$$\text{Maximise } Z = 3x_1 - x_2$$

Subject to the constraints

$$2x_1 + x_2 \geq 2$$

$$x_1 + 3x_2 \leq 3$$

$$x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

9. Write the algorithm for solving LPP problem by using Simplex method. 10

Answer any two of the following questions:

OPE-006
10 × 2 = 20

10. A project schedule has the following characteristics.

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Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Time(days)	2	2	1	4	8	5	3	1	5	4	3

Construct the PERT network and find the critical path and time duration of the project.

11. Solve the following transportation problem.

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	D ₁	D ₂	D ₃	D ₄	Supply
O ₁	3	8	9	16	8
O ₂	6	11	14	9	9
O ₃	5	13	10	12	13
Demand	6	7	7	10	30

12. Solve the cost minimizing assignment problem.

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Jobs/Machines	Machine A	Machine B	Machine C	Machine D	Machine E
Job1	11	10	18	5	9
Job2	14	13	12	19	6
Job3	5	3	4	2	4
Job4	15	18	17	9	12
Job5	10	11	19	6	14
